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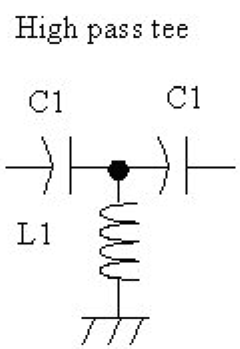
Low Pass Pi-Network Phase Shifter:

References: Microwave Engineering, Pozar.

ABCD matrices can be used to represent:

- transmission line with a characteristic impedance of Zo and phase shift of ϕ
- lumped equivalent phase shift networks.

By equating these two matrices, the values of L's and C's in the lumped network can be determined. This method is shown for four lumped phase shifters.



The ABCD matrix for this network can be written as:

$$\begin{bmatrix} 1 & \frac{1}{j\omega C_1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{j\omega L_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{j\omega C_1} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{1}{L_1 C_1 \omega^2} & \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1}\left(1 - \frac{1}{L_1 C_1 \omega^2}\right) \\ \frac{1}{j\omega L_1} & 1 - \frac{1}{L_1 C_1 \omega^2} \end{bmatrix}$$

Next write an ABCD matrix for a transmission line of characteristic impedance Zo and phase shift of ϕ.

$$\begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

Equate the two matrices and solve for L1:

$$\begin{bmatrix} 1 - \frac{1}{L_1 C_1 \omega^2} & \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1}\left(1 - \frac{1}{L_1 C_1 \omega^2}\right) \\ \frac{1}{j\omega L_1} & 1 - \frac{1}{L_1 C_1 \omega^2} \end{bmatrix} = \begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

$$\frac{1}{j\omega L_1} = jY_o sin\phi$$

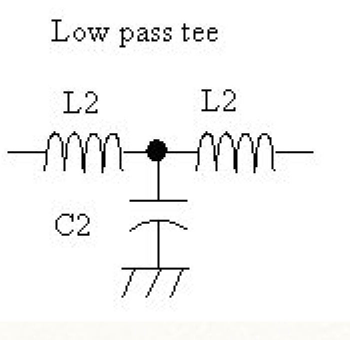
$$L_1 = \frac{1}{\omega Y_o sin\phi} = \frac{Z_o}{\omega sin\phi}$$

Substitute L1 into alternate matrix entry and solve for C1.

$$1 - \frac{1}{L_1 C_1 \omega^2} = \cos\phi$$

$$1 - \frac{1}{\frac{Z_o}{\omega sin\phi} C_1 \omega^2} = \cos\phi$$

$$C_1 = \frac{sin\phi}{\omega Z_o (1 - \cos\phi)}$$



The ABCD matrix for this network can be written as:

$$\begin{bmatrix} 1 & j\omega L_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & j\omega L_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - L_2 C_2 \omega^2 & j\omega L_2 (1 - L_2 C_2 \omega^2) + j\omega L_2 \\ j\omega C_2 & 1 - L_2 C_2 \omega^2 \end{bmatrix}$$

Next write an ABCD matrix for a transmission line of characteristic impedance Zo and phase shift of ϕ.

$$\begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

Equate the two matrices and solve for C2:

$$\begin{bmatrix} 1 - L_2 C_2 \omega^2 & j\omega L_2 (1 - L_2 C_2 \omega^2) + j\omega L_2 \\ j\omega C_2 & 1 - L_2 C_2 \omega^2 \end{bmatrix} = \begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

$$j\omega C_2 = jY_o sin\phi$$

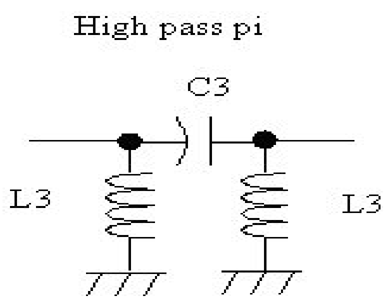
$$C_2 = \frac{Y_o sin\phi}{\omega} = \frac{sin\phi}{\omega Z_o}$$

Substitute C2 into alternate matrix entry and solve for L2.

$$1 - L_2 C_2 \omega^2 = \cos\phi$$

$$1 - L_2 \frac{sin\phi}{\omega Z_o} \omega^2 = \cos\phi$$

$$L_2 = \frac{Z_o(1 - \cos\phi)}{\omega sin\phi}$$



The ABCD matrix for this network can be written as:

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{j\omega L_3} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{j\omega C_3} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{j\omega L_3} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{1}{L_3 C_3 \omega^2} & \frac{1}{j\omega C_3} \\ \frac{1}{j\omega L_3} + \frac{1}{j\omega L_3}\left(1 - \frac{1}{L_3 C_3 \omega^2}\right) & 1 - \frac{1}{L_3 C_3 \omega^2} \end{bmatrix}$$

Next write an ABCD matrix for a transmission line of characteristic impedance Zo and phase shift of ϕ.

$$\begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

Equate the two matrices and solve for C3:

$$\begin{bmatrix} 1 - \frac{1}{L_3 C_3 \omega^2} & \frac{1}{j\omega C_3} \\ \frac{1}{j\omega L_3} + \frac{1}{j\omega L_3}\left(1 - \frac{1}{L_3 C_3 \omega^2}\right) & 1 - \frac{1}{L_3 C_3 \omega^2} \end{bmatrix} = \begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

$$\frac{1}{j\omega C_3} = jZ_o sin\phi$$

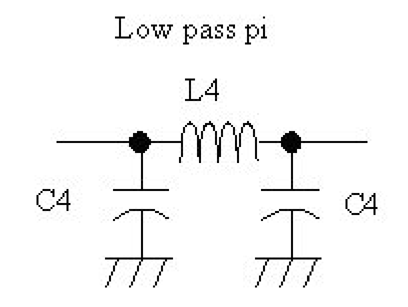
$$C_3 = \frac{1}{\omega Z_o sin\phi}$$

Substitute C3 into alternate matrix entry and solve for L3.

$$1 - \frac{1}{L_3 C_3 \omega^2} = \cos\phi$$

$$1 - \frac{1}{L_3 \frac{1}{\omega Z_o sin\phi} \omega^2} = \cos\phi$$

$$L_3 = \frac{Z_o sin\phi}{\omega(1 - \cos\phi)}$$



The ABCD matrix for this network can be written as:

$$\begin{bmatrix} 1 & 0 \\ j\omega C_4 & 1 \end{bmatrix} \begin{bmatrix} 1 & j\omega L_4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C_4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - L_4 C_4 \omega^2 & j\omega L_4 \\ j\omega C_4 + j\omega C_4 (1 - L_4 C_4 \omega^2) & 1 - L_4 C_4 \omega^2 \end{bmatrix}$$

Next write an ABCD matrix for a transmission line of characteristic impedance Zo and phase shift of ϕ.

$$\begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

Equate the two matrices and solve for L4:

$$\begin{bmatrix} 1 - L_4 C_4 \omega^2 & j\omega L_4 \\ (j\omega C_4)^2 (1 - L_4 C_4 \omega^2) & 1 - L_4 C_4 \omega^2 \end{bmatrix} = \begin{bmatrix} \cos\phi & jZ_o sin\phi \\ jY_o sin\phi & \cos\phi \end{bmatrix}$$

$$j\omega L_4 = jZ_o sin\phi$$

$$L_4 = \frac{Z_o sin\phi}{\omega}$$

Substitute L4 into alternate matrix entry and solve for C4.

$$1 - L_4 C_4 \omega^2 = \cos\phi$$

$$1 - \frac{Z_o sin\phi}{\omega} C_4 \omega^2 = \cos\phi$$

$$1 - Z_o sin\phi C_4 \omega = \cos\phi$$

$$C_4 = \frac{1 - \cos\phi}{\omega Z_o sin\phi}$$